Philosophy 211
Sample In-Class Exam 2

I. Complete these proofs. There are no additional assumptions. (14)

A. ∃xPx → ∃xQx, ∀x~(Qx & Sx)
├ ∀x(Px → ∃y~Sy)

1	(1) ∃xPx → ∃xQx	A
2	(2) ∀x~(Qx & Sx)	A
3	(3) Pa	A
____	(4) ∃xPx	_______
____	(5) ∃xQx	_______
6	(6) Qb	A
____	(7) ~(Qb & Sb)	_______
____	(8) ~Qb v ~Sb	_______
____	(9) ~Sb	_______
____	(10) ∃y~Sy	_______
____	(11) ∃y~Sy	_______
____	(12) Pa → ∃y~Sy	_______
____	(13) ∀x(Px → ∃y~Sy)	_______

B. ∃y∀xRxy, ∃x∀yRxy,
├ ∀z∃x∃y(Rxz & Rzy)

1	(1) ∃y∀xRxy	A
2	(2) ∃x∀yRxy	A
3	(3) ∀xRxa	A
4	(4) ∀yRby	A
____	(5) Rca	_______
____	(6) Rbc	_______
____	(7) Rbc & Rca	_______
____	(8) ∃y(Rbc & Rcy)	_______
____	(9) ∃x∃y(Rxc & Rcy)	_______
____	(10) ∃x∃y(Rxc & Rcy)	______
____	(11) ∃x∃y(Rxc & Rcy)	______
____	(12) ∀z∃x∃y(Rxz & Rzy)	__

II. Find the errors in each of these proofs and explain why they are errors. (10)

A. ∀x∃y(Px → Qy) ├ ∀xPx → ∀yQy

1	(1) ∀x∃y(Px → Qy)	A
2	(2) ∀xPx	A
1	(3) ∃y(Pa → Qy)	1 ∀E
4	(4) Pa → Qb	A
2	(5) Pa	2 ∀E
2,4	(6) Qb	4,5 →E
2,4	(7) ∀yQy	6 ∀I
1,2	(8) ∀yQy	3,7 ∃E(4)
1	(9) ∀xPx → ∀yQy	8 →I(2)

B. $\forall x(Dx \rightarrow Ax) \vdash \forall x(\exists y(Hxy \,\&\, Dy) \rightarrow \exists z(Hxz \,\&\, Az))$

1	(1) $\forall x(Dx \rightarrow Ax)$	A
2	(2) Hab & Db	A
2	(3) Hab	2 &E
2	(4) Db	2 &E
1	(5) $Db \rightarrow Ab$	3 &E
1,2	(6) Ab	4 &E
1,2	(7) Hab & Ab	3,6 &I
1,2	(8) $\exists z(Haz \,\&\, Az)$	7 $\exists$I
1	(9) $(Hab \,\&\, Db) \rightarrow \exists z(Haz \,\&\, Az)$ 8$\rightarrow$I(2)	
1	(10) $\exists y(Hay\& \, Dy) \rightarrow \exists z(Haz \,\& Az)$ 9$\exists$I	
1	(11) $\forall x(\exists y(Hxy \,\&\, Dy) \rightarrow \exists z(Hxz \,\&\, Az))$ 10 $\forall$I	

III. Paraphrase the following English sentences into Predicate Logic using the following translation scheme: (25)

$A\alpha$: α is on Team A
$B\alpha$: α is on Team B
$D\alpha\beta$: α defeated β
m: Mary
t: Tom

1. Tom did not defeat everyone on Team A who was defeated by Mary.

2. If no one on Team A defeated Tom, then there is someone on Team A who did not defeat Mary.

3. Every member of Team A who defeated Mary was defeated by at least one member of Team B. (NOTE: I intend this to allow that different members of Team A who defeated Mary may have been defeated by different members of Team B.)

4. If anyone on Team A defeated anyone on Team B, then everyone on Team B was defeated by Mary.

5. Tom defeated at most one member of Team A.

III. Determine whether these sentences are true on the given diagrams: (16)

	D1	D2	D3	D4
∀x(Mx → ∃y(Ty & Ayx))				
∃x(Mx & ∀y(Ty → Ayx))				
∃x(Tx & ∀y(My → ~Axy))				
∀x(Tx → ∃y∃z(My & Mz & y ≠ z & Axy & Axz))				

Diagram 1

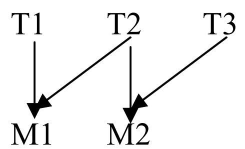


Diagram 2

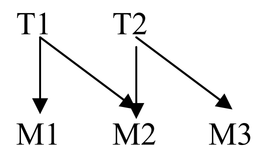


Diagram 3

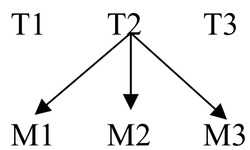
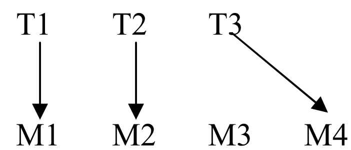


Diagram 4



Prove these sequents: (35)

∀x∀y(~Rxy → Ryx), ∀x∀y(Rxy → Ryx) ├ ∀x∀yRxy

∀x(Px → ~Qx), ∃x(Sx & Qx) ├ ∃x(Sx & ~Px)

∃x∀yRxy, ∀x∀y(Rxy → ~Ryx) ├ ∀x∃y~Rxy

∃x∀yRxy, ∀x((Px & ∃yRyy) → ∀y~Ryx) ├ ∀x~Px